Reply to Referee3

Text coloring: original Referee3 text, our non-manuscript comments, our in-manuscript text.

We appreciate a careful evaluation of our work by Referee3.

Journal of Fluid Mechanics – JFM-2024-1620 – Review

In the present manuscript, the authors present the results of permeability simulations using discrete superstructures with the goal to minimize the discretization error in simulations of flow through a porous medium. The paper is well organized and could be improved with the following comments addressed:

- **General:** Low or high resolution are relative terms. 1. What is considered "low" and "high" resolution for the purposes of this study? Please elaborate. 2. Also, suggest using only two terms for resolution throughout document (i.e., low and high resolution or coarse and fine resolution) and 3. avoid using increase and decrease as these terms are relative.
    - We added the following text*:* In this study, we deal with the packings of closely packed spheres discretized on a uniform cubic mesh. We refer to the discretization resolutions of below ~10 voxels per sphere diameter as "low", while resolutions of above ~50 voxels per sphere diameter are considered as "high". If we consider the hydraulic diameter d_h to be about one third of a sphere diameter for a packing with porosity of 0.35 (Whitaker1972, Cengel2013), then the low resolutions will correspond to $d_h \lesssim 3$ voxels while high resolutions to $d_h \gtrsim 15$.
    - We re-checked and did not find any usage of "coarse" or "fine" terms except the bibliography, where the title of a previous study includes these terms.
    - In some cases, we need to speak not about particular resolution range but about change of the resolution in given direction (for example, what happens with the discretization error when resolution change from one range to another). We modified the following sentences:
        - Line 142: Conversely, with increasing resolution from low to high values the superstructures disappear and flow occurs through the pores of the underlying analytical geometry.
        - Line 204: Note that in Figure4A increase of the resolution from low to high result in all curves …
        - Line 332: with decreases in resolution to low values.
        - Line 338: By contrast, increases in resolution from low to high values results in …
- **Lines 104-106:** Add the reference values and add citations for them. We appreciate that this information was requested by Referee3. The reference values were present initially, but they were removed due to the length limitation of the "JFM Rapids". Now they are available in Appendix, Tables A1 and A2.
- **Figure 3:** Should U = 4, be U = 3 or was U = 4 also tested? And if it was tested, it would be helpful to mention this somewhere in the text. Also, adding a colorbar for the magnitude of the absolute velocity would be helpful. While "blue" is low and "red" is high velocity, the remaining colors and what magnitude they represent is not clear. There was no intention to hide flow fields at U = 3, and we agree that the verbal description of the colorbar is definitely not the best solution. Therefore, motivated by this comment, we significantly extended Figure 3 in quantitative direction by adding universal colorbar

and tables with the colorbar limits. Also, we added appendix Figure 1A to show the cases of U = 4 and U = 5. We decided to not show U = 2, 3, 4, 5 in one figure to avoid making the figure too large or its panels too small.

- **Line 182-184:** It is stated "Note that in Figure 4A increases in resolution result in all curves converging to zero from above, crossing the zero error value, and then slowly continuing to converge up from below." However, the results for magic number = 3/8 (green) stops at the null point (does not cross the zero error or converge up). Please clarify. There is nothing special about the case of Λ = 3/8. It is the same as other two cases: with the resolution increase above 30 the error will cross 0 and become negative. We added the following text and the reference, where this situation is actually illustrated: (This also includes Λ = 3/8 in Figure 4A, as can be seen in Figure 8c,d in (Khirevich et al. 2015).)
- **Lines 231-232:** For the two laboratory-packed CT scanned geometries, what was the diameter of the packing material, was the same material used for both packed geometries, what was the scanning resolution and other relevant scanning information (exposure time, kV, projections). What kind of device was used for scanning? We appreciate for requesting these details. We extended the details and references to our CT scanning procedure, which is now 2nd paragraph in Section 6.
- **Lines 237-239:** Elaborate (add a sentence or two) on why the maldistribution is significant for the ratio of 10 sphere diameters. We added/modified the following text: "propagating 3–5 diameters from the wall into the bulk (see porosity and velocity profiles in (Khirevich et al. 2007), Figures 2 and 4). The impact of the confining wall is significant for the ratio of 10 sphere diameters per cylindrical container, and therefore this geometry is also used to assess eq. (5.4)."
- **Section 6**: In certain situations, faster scans/lower resolutions are better (e.g., imaging a part of the human body) while longer scans at higher resolutions are better for other scenarios (e.g., fluid flow in pore structures). While increasing the resolution of a scan can lead to a longer scan time (and larger datasets which are cumbersome), if essential pore structure, specifically pore connectivity features, are below the scanning resolution, the fluid simulations will be missing fundamental information. **A**. What type of sandstone was used in the X-ray scans and **B**. was the sample scanned at different resolutions (specifically, finer resolutions) for comparison? If not, adding the results from more than one scanning resolution to validate the results would be useful. **A**. We extended the explanation and references to our experimental packing procedures to pack the samples P3 and P4, which is now 2nd paragraph in Section 6. **B**. Each sample was scanned not at one, but at 18 separate resolutions (from 3 to 64 voxels per sphere diameter). I.e., there are 18 different CT images, which are also available online. We added this information, mostly in 2nd paragraph of Section 6.
- **Figure 5**: (1) Define and explain what P3 and P4 are in the text, in addition to the figure title; (2) for consistency, adjust y-axis of A and B to both be 0 to 200; (3) also define PDF in the subplot in C; and (4) in Figure 3C scalebars should be added. We adapted the article content to requests (1), (2), (4), while for (3) we renamed "PDF" to "frequency".
- **Suggestions to improve overall clarity**:
    - **General**: 15 percent = 15 %; suggest changing throughout. We did not find the word "percent" in our text, and use symbol "%" through the text. Otherwise, this comment is not clear.
    - **General**: fluid and solid typeface deviates from the font used for main text used throughout

the document - correct to be a consistent typeface. We intentionally use `teletype`, `monospaced` font to refer to `solid` voxels. The normal typesetting is used to refer to other context, such as solid wall, solid boundary. Hence, we prefer to keep `monospaced` typesetting to have emphasis on the reference to `solid` or `fluid` voxels. This is in consistency with our previous experimental work doi: 10.1063/5.0123673.

- **Figure 1 Caption**: “while the middle and right columns — non-integer L/U ratios.” Is not acomplete sentence. Please correct. We changed the text as requested.
- **Line 31**: can be used to verify laboratory results. Corrected.
- **Lines 55**: define CT - computed tomography (CT) – then use throughout document. Corrected as requested.
- **Line 111**: reduces the scatter of the Corrected.
- **Line 126**: define NB It is from Latin “nota bene”. But we removed this abbreviation.
- **Line 224**: provide should be provides Corrected.
- **Line 276**: increases in resolution results... should be result Corrected, thank you.

11-Nov-24, message from editors:

Referee: 3

Comments to the Author

All my comments have been addressed.